Time-Optimal Control of an Ultrasonic Sensor

Kartik V. Sastry

Student, Electrical and Computer Engineering

Teammate: Aravind Balasubramanian (ECE)

Research Advisor: Prof. David Taylor (ECE)

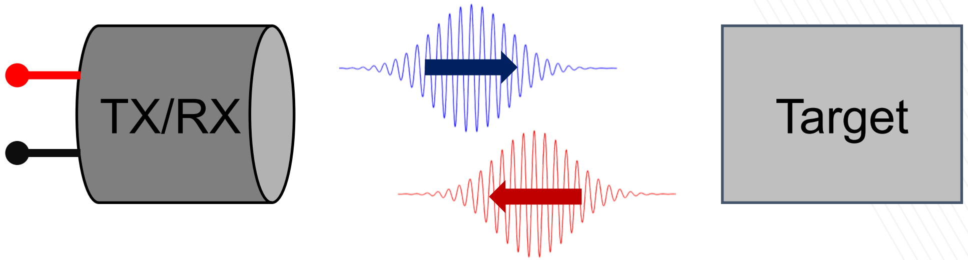
Project Sponsor: Dr. David Magee (Texas Instruments)

Outline

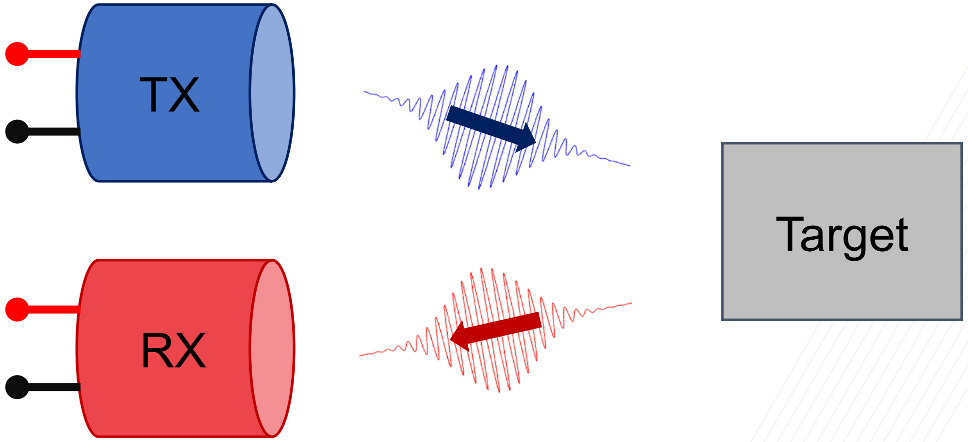
- Operating principles of ultrasonic distance sensors
- Range and resolution limitations of ultrasonic distance sensors
- Improving sensor range and resolution via time-optimal control
- Role of implementation constraints in formulation of the control problem
- Model-free solution to implementation-constrained time-optimal control problem

Distance Measurement with an Ultrasonic Sensor

- An ultrasonic sensor consists of one or more ultrasonic transducers and a signal processor
- Distance measurement begins with the transmission of a pulse towards a target
- Under suitable conditions, an echo is received after a time delay
  - Target orientation, material composition and transmit pulse strength play major roles
- Transducer voltage waveforms are processed to estimate the time delay and compute distance to the target



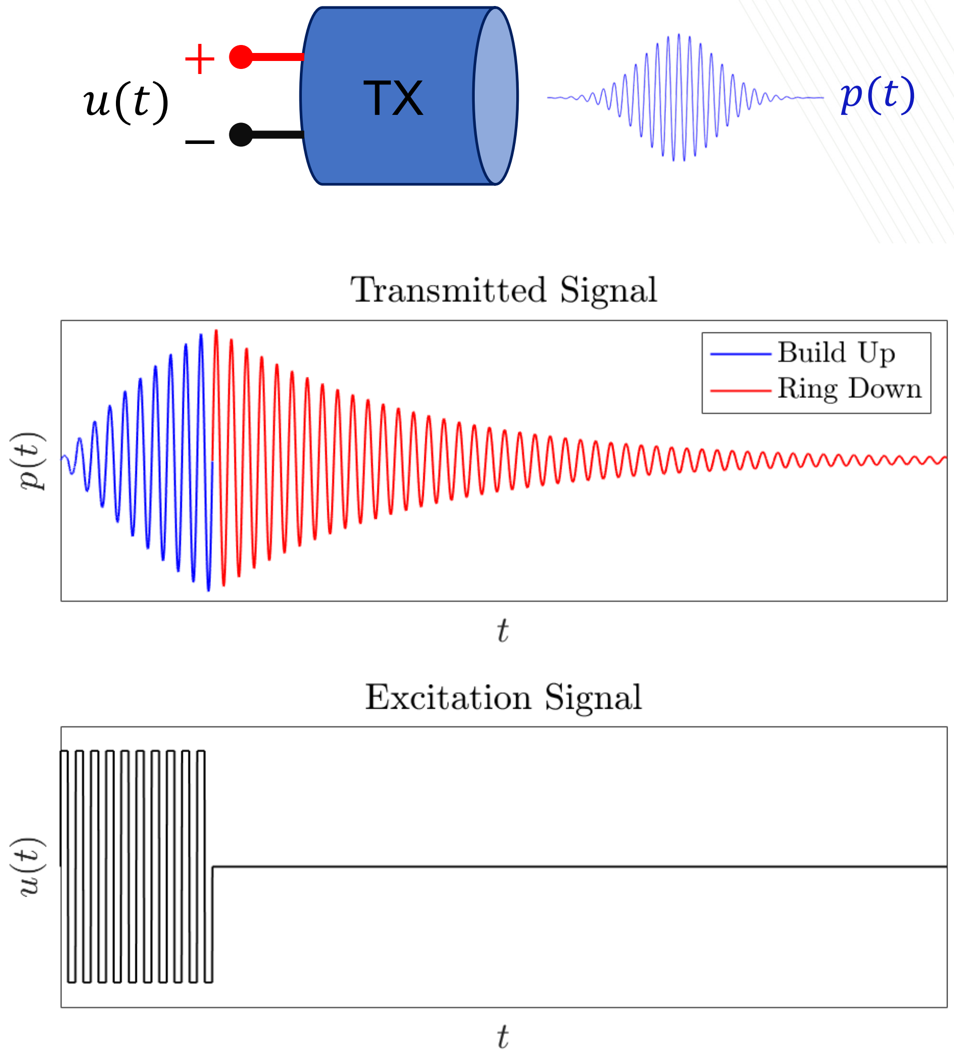
Single-transducer configuration (monostatic)



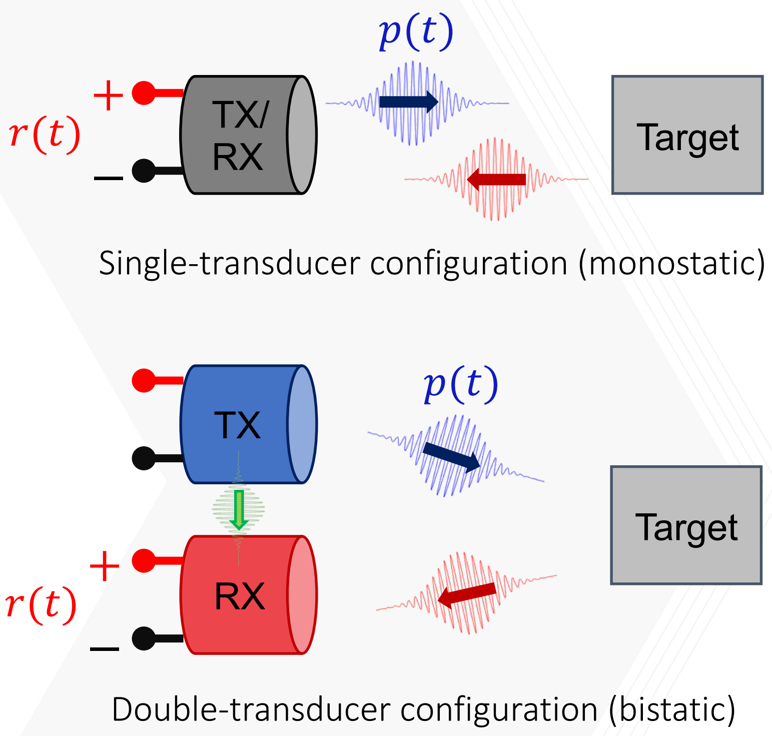
Double-transducer configuration (bistatic)

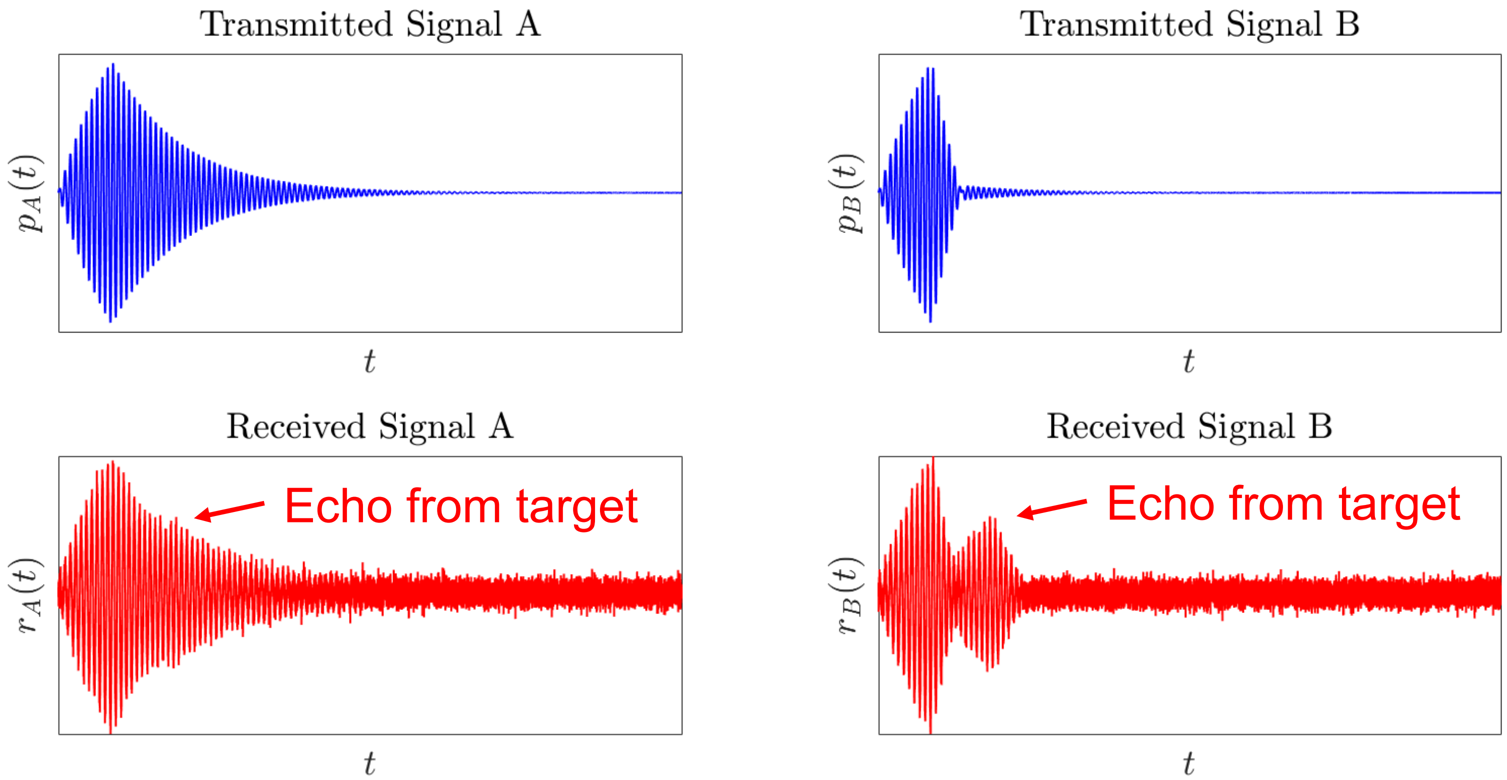
Pulse Generation in Piezoelectric Ultrasonic Transducers

- Piezoelectric ultrasonic transducers are poorly damped resonant systems by design
- Typical excitation signal consists of an integer number of pulses at the resonant frequency
  - Low damping ensures high-amplitude pressure wave transmission
    - Necessary to sense distant targets and combat propagation/reflection loss
  - Low damping gives rise to a long ring-down period
    - Ringing has several undesirable consequences
- *Transducer design tradeoff:* Introducing damping decreases ring-down time at the cost of pulse strength



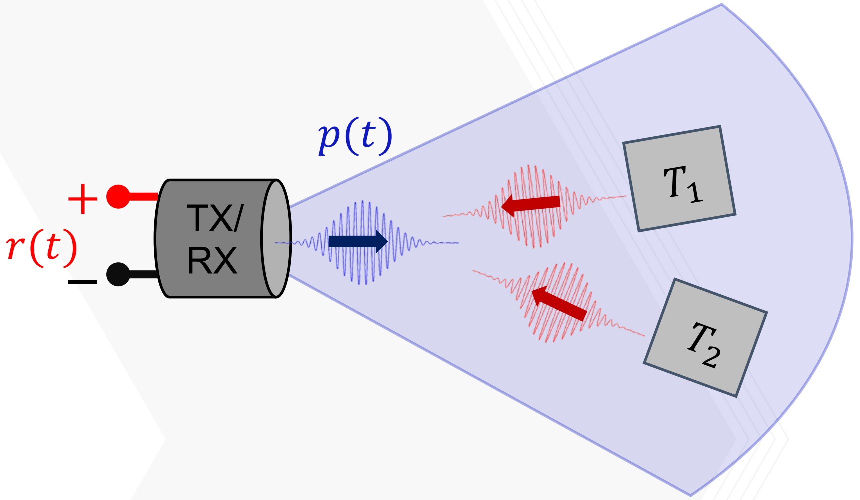
Issue #1: Ringing Limits Sensing Range



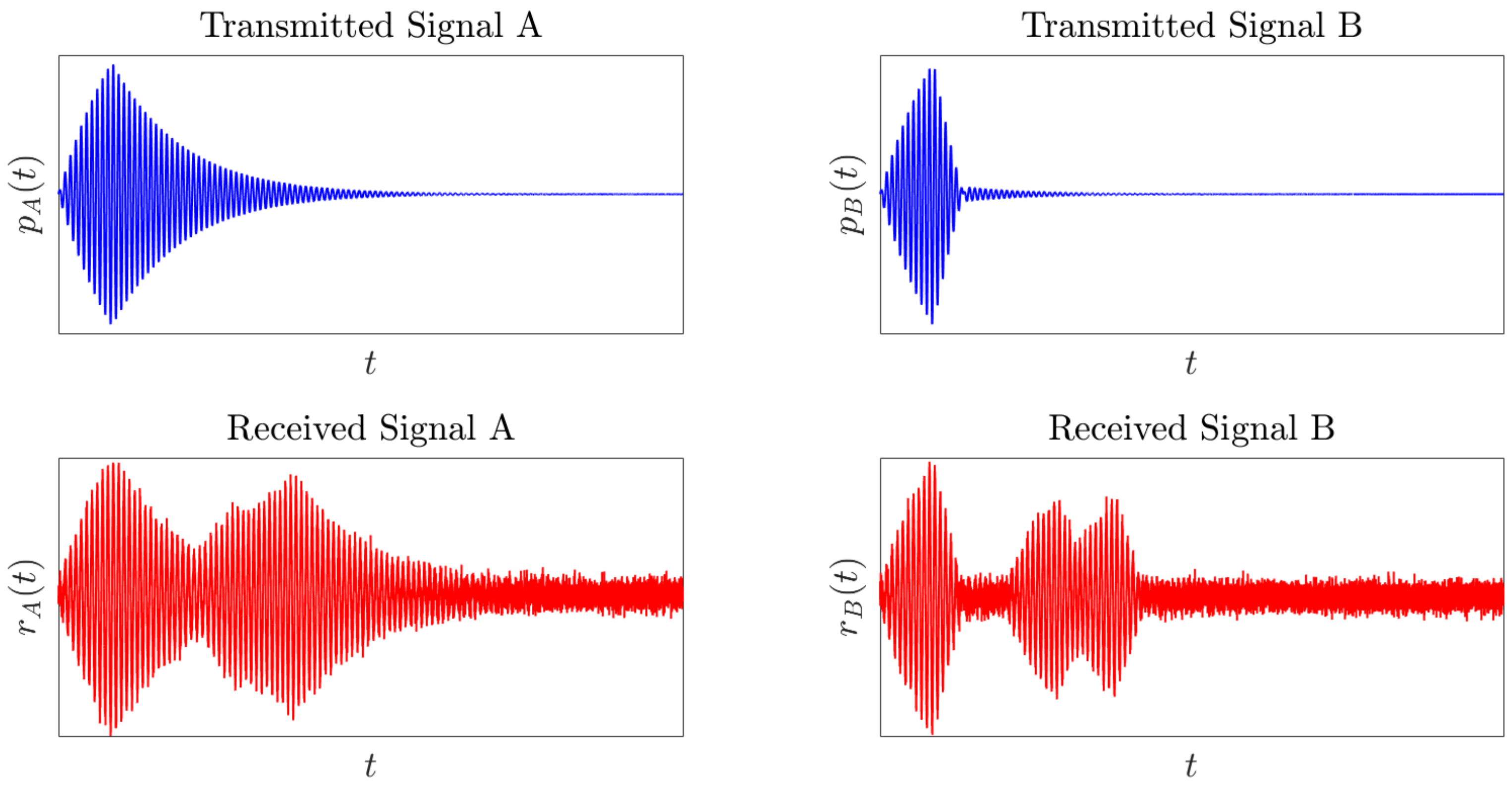


- *Scenario:* Object located close to sensor
- Without ringing suppression, *the received echo is completely obscured by ringing*
  - Clearly, both thresholding and correlation-based detection schemes will fail here
- Note: Transmit activity can appear in $r_{A/B}(t)$ even in the bistatic case due to *crosstalk (green)*

Issue #2: Ringing Limits Sensing Resolution

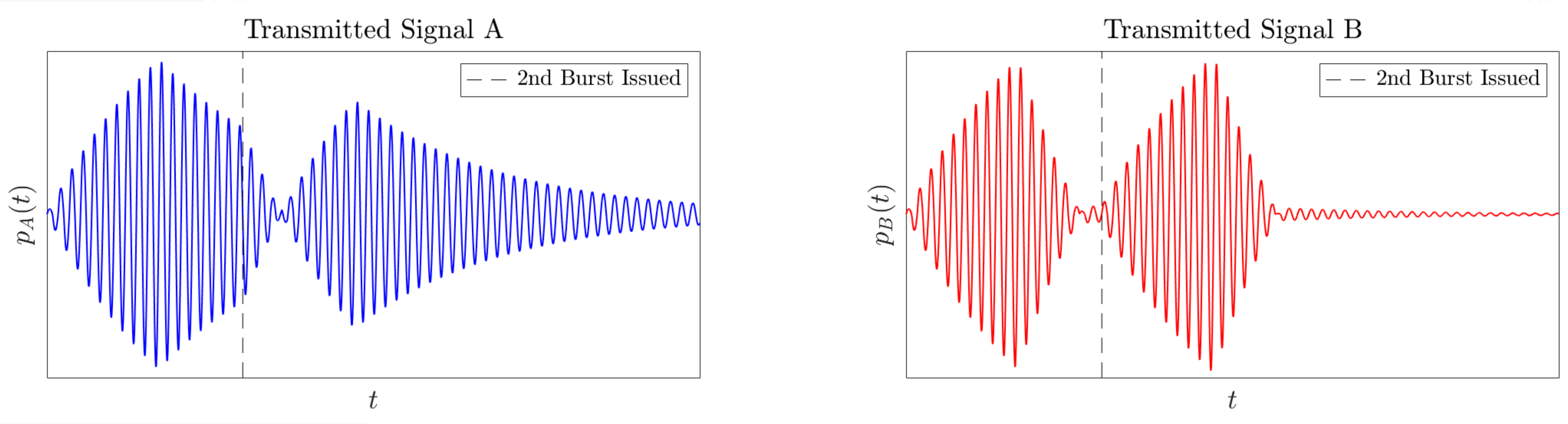


Monostatic configuration shown. Applies equally to bistatic configuration



- *Scenario:* Two distinct targets located close together within field of view
- Transducers have a finite *beam width*. Acoustic energy is transmitted into a conical region of space which may contain multiple objects. Multiple echoes may return
- Without ringing suppression, the *echoes from target T_1 and target T_2 are not separable*

Issue #3: Ringing Creates Variability in Pulse Transmission



- *Scenario:* Two burst events issued in rapid succession
- This situation could arise if multiple distance measurements are being averaged to reduce the influence of noise
- The amplitude of the second burst pulse can vary significantly depending on the time separation between burst events
  - Can lead to failed object detections when detections are expected

First Statement of the Control Problem

- Poor damping of piezoelectric ultrasonic transducers is beneficial during pulse transmission, but also gives rise to undesirable ringing
  - We wish to drive the system to rest in minimum time, from its state at the *end of burst*
- The natural solution is time-optimal control. May consider continuous-time first
- Given x_0, choose $u(t) \in \mathcal{U}_c$ for $0 \leq t \leq T$ to

$$\begin{aligned} \text{minimize} \quad & T \\ \text{subject to} \quad & \dot{x}(t) = Ax(t) + Bu(t) \\ & x(0) = x_0, \qquad x(T) = 0 \end{aligned}$$

- Possible input constraint set: $\mathcal{U}_c = \{u : |u| \leq U_{max}\}$
- This problem is treated in textbooks, solution is bang-bang with edges in $[0, T]$ [1]
- Requires analog circuitry to implement. We favor microcontroller-based implementation

Georgia Tech
CREATING THE NEXT

Reformulating the Control Problem

- If using a microcontroller, placement of edges is restricted to a discrete set of points. Discrete-time time-optimal control is more appropriate:
- Given x_0, choose $u[k] \in \mathcal{U}_d$ for $k = 0, \dots, K-1$ to

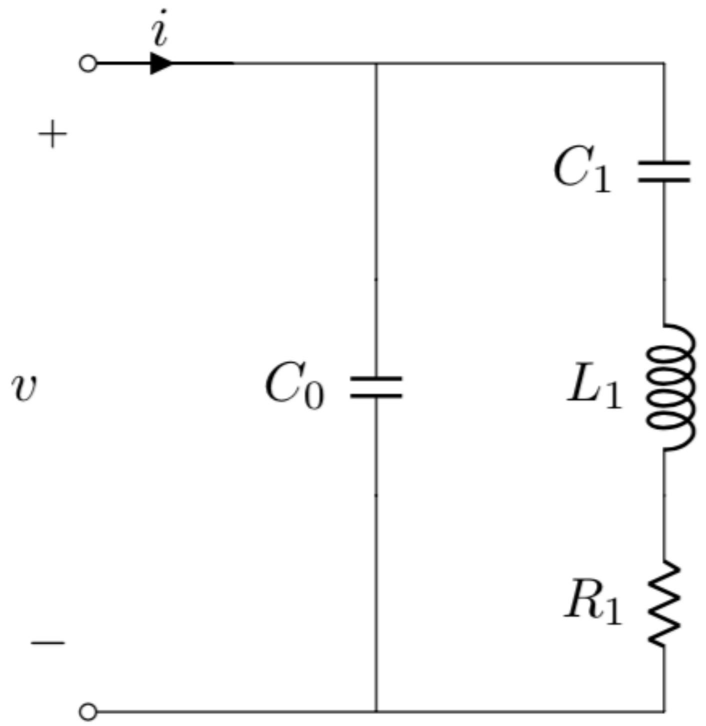
$$\begin{aligned} \text{minimize} \quad & K \\ \text{subject to} \quad & x[k+1] = A_d x[k] + B_d u[k] \\ & x[0] = x_0, \qquad x[K] \in \mathcal{S} \end{aligned}$$

- Connections between continuous and discrete problems: $A_d = e^{AT_s}, \quad B_d = \int_0^{T_s} e^{At} B \, dt, \qquad T \sim KT_s$
- $\mathcal{U}_d = \{u : |u| \le U_{max}\}, \; \mathcal{S}_0 = \{0\}$
  - Solution via convex optimization in [2]; does not account for actuator quantization
- $\mathcal{U}_d = \{\pm(q/Q)U_{max}, \; q = 0, 1, \dots, Q\}, \; \mathcal{S}_\epsilon = \{x : \|x\| \le \epsilon\}$
  - Accounts for discrete-time and discrete signal amplitude constraints [3]
  - $\mathcal{U}_\mathrm{d}$ is a finite set, therefore searchable. Special case: $Q = 1 \Rightarrow \mathcal{U}_d = \{-U_{max}, 0, U_{max}\}$
  - Search-based solution allows us to shed dependence on a model

Georgia Tech
CREATING THE NEXT

Inspiration: Time-Optimal Control of the Harmonic Oscillator

- Could exhaustively search over all possible control sequences such that $u[k] \in \{-U_{max}, 0, U_{max}\},\ k = 0, 1, \ldots K-1$
- *Can we use intuition to find a more intelligent (faster) solution?*
- Transducer response resembles that of a second-order damped harmonic oscillator
  - Due to *poor damping*, studying the *undamped* oscillator is informative
- The continuous-time time-optimal control to drive the state of the harmonic oscillator from an initial condition to the origin is bang-bang at the resonant frequency
  - Easy to realize (at least approximately) using a microcontroller
  - Reduces size of search space!



Butterworth Van-Dyke model of an ultrasonic transducer – *RLC* branch represents a mechanical oscillator

Inspiration: Time-Optimal Control of the Harmonic Oscillator

- The undamped harmonic oscillator is described by:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & \omega\\ -\omega & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \qquad , |u(t)| \leq 1$$

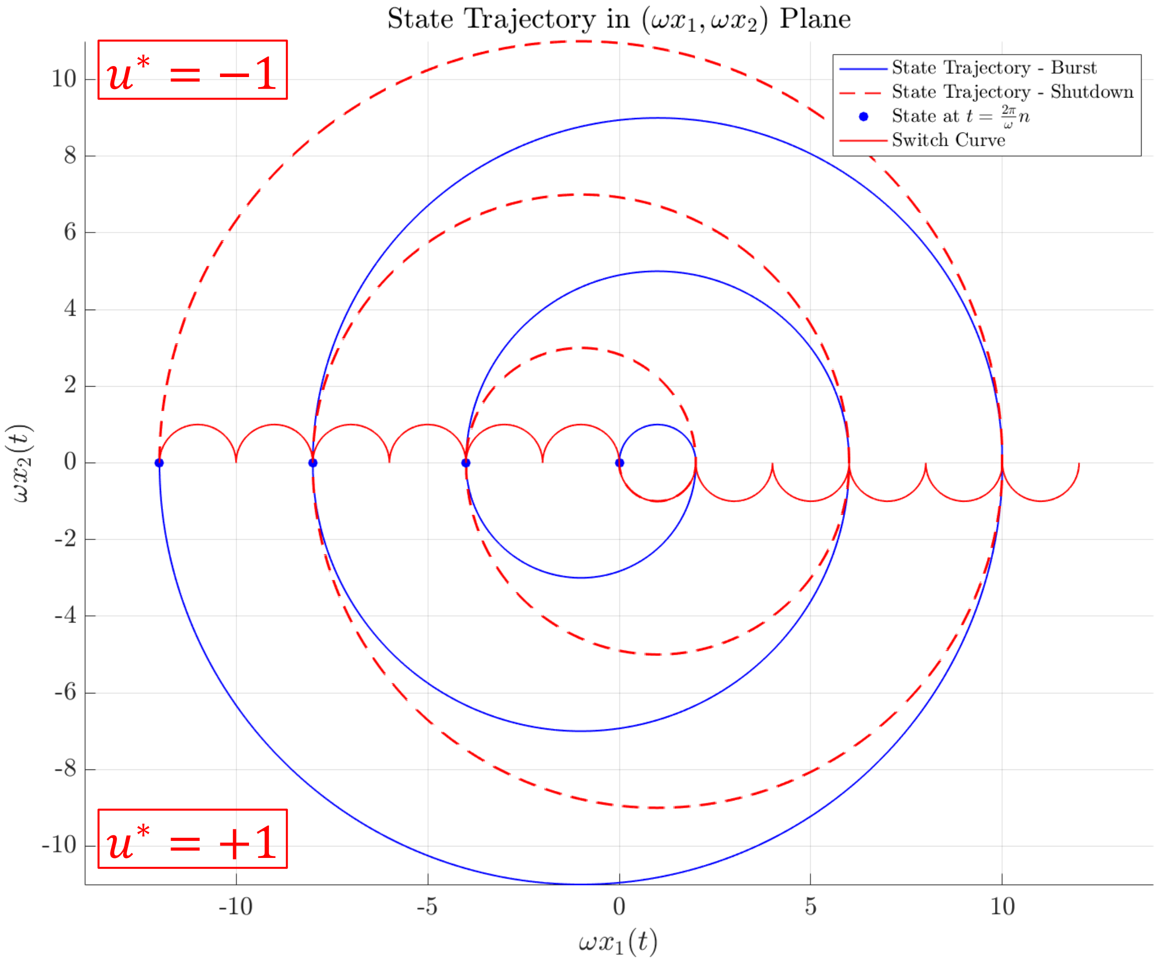
- It can be shown that when subjected to the input:

$$u(t) = \begin{cases}\text{sign}(\sin(\omega t)) & \text{, for } N_E \text{ cycles}\\ 0 & \text{, otherwise}\end{cases}$$

- The (scaled) state after the n^{th} of N_E cycles is given by:

$$\omega x\left(\frac{2\pi}{\omega}n\right) = \begin{bmatrix}-4n\\ 0\end{bmatrix} \quad , n = 0, \ldots N_E$$

- The time origin for control is the end of burst; initial conditions for control are located on the switch curve
- The first control action must therefore be -1 for π/ω seconds, irrespective of the resonant frequency!



Algorithm: Model-Free Shutdown

- We propose finding the time-optimal control, subject to implementation constraints, via a brief, one-time power-on calibration procedure:

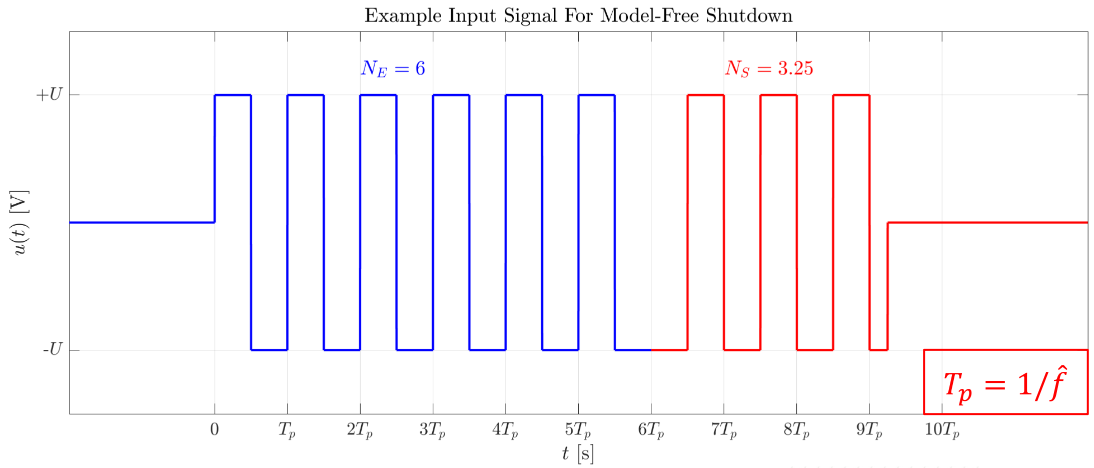
$\hat{f}$ = *Estimated resonant frequency in Hz*
$N_{s,prev}$ = 0
for N_E = 1 : 10
    t_{min} = $+\infty$
    for N_s = $N_{s,prev}$: 1/16 : N_E
        Issue N_E *burst pulses at* $\hat{f}$ *Hz*
        Issue N_s *shutdown pulses at* $\hat{f}$ *Hz*
        t_s = *Measured settling time*
        if $t_s < t_{min}$
            t_{min} = t_s
            $N_{s,prev}$ = N_s
        end if
    end for
    Record $N_{s,prev}$ *in* ***Table***
end for

- In the worst case (if $N_S = 0$ always recommended), 890 burst-and-measure trials are conducted
- If the sample table on the right is produced, 370 burst-and-measure trials are conducted

Sample Table Constructed by Algorithm

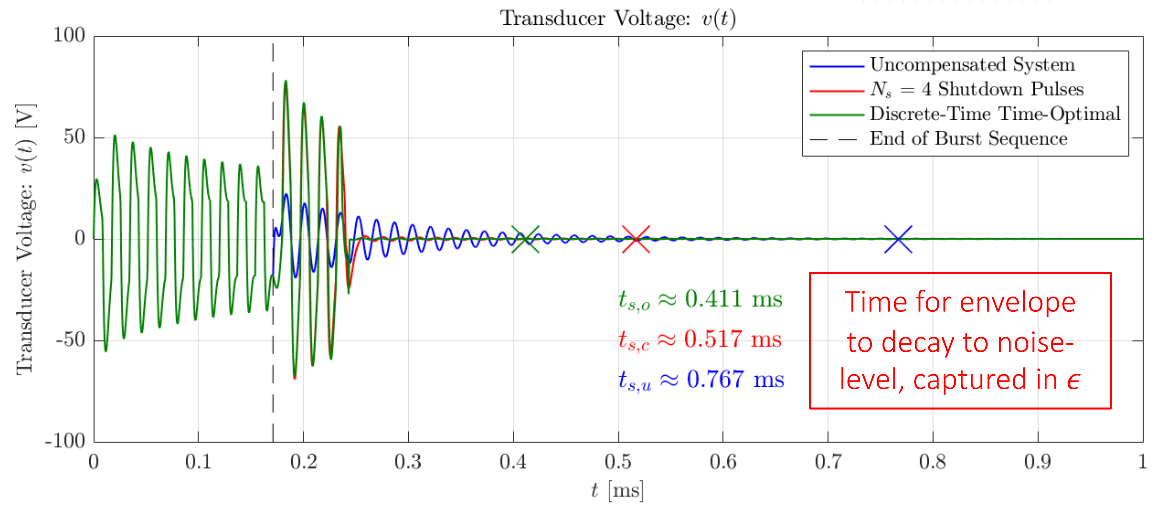
N_E	N_s	N_E	N_s
1	2	6	4
2	2.5	7	4.5
3	2.5	8	5
4	3	9	5.5
5	3.5	10	5.5

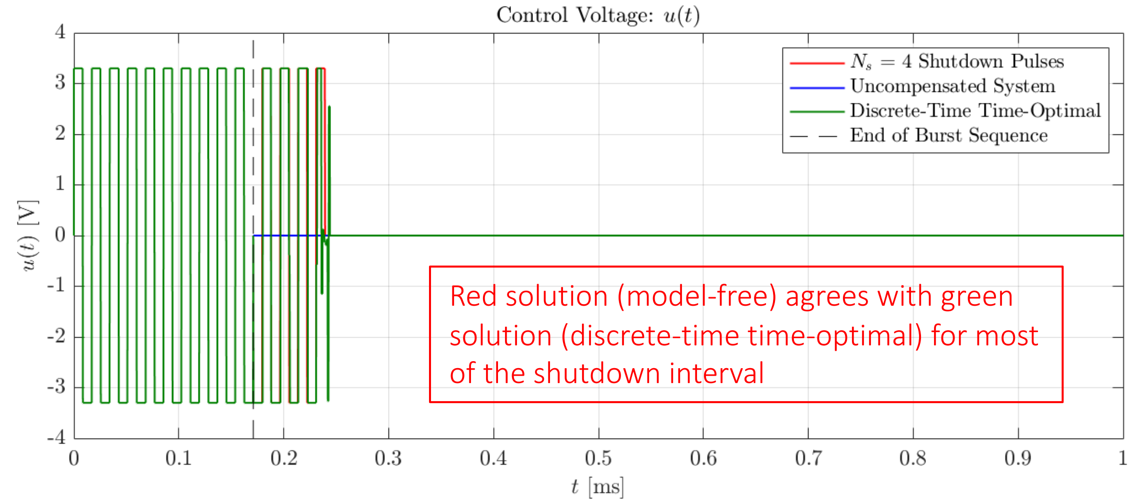
Sample Candidate Excitation Sequence



Results and Comparison: Model-Free Shutdown

- Simulation model developed for a 58 kHz transducer + impedance matching network
- **Blue:** No shutdown action
- **Green:** Discrete-time time-optimal shutdown
- **Red:** Model-free shutdown
  - $\mathcal{U}_d = \{-U_{max}, 0,\ U_{max}\},\ \mathcal{S}_\epsilon = \{x :\ \|x\| \leq \epsilon\}$
- Discrete-time time-optimal solution provides a *benchmark*
  - A precise model and a departure from current hardware are required for implementation
- Table construction is done once at power-up. At runtime, only a table *lookup* is required. No expensive computation!





Georgia Tech
CREATING THE NEXT

Summary, Conclusion

- To enhance sensor range and resolution, short pulses are preferred over long pulses
- Time-optimal control is employed to produce the *shortest possible* pulses by suppressing ringing after burst
- Practical implementation constraints influence the formulation of the control problem
- A simple, model-free solution to the properly reformulated control problem is presented
    - This solution is *time-optimal,* subject to practical implementation constraints
    - This solution is model-free. Not susceptible to model parameter error
    - This solution is inspired by 2nd order theory, but was shown to perform well even on a 4th order system

Georgia Tech
CREATING THE NEXT

References

[1] M. Athans and P. Falb, *Optimal Control: An Introduction to the Theory and Its Applications*. McGraw-Hill, 1966.

[2] L. Bako, D. Chen, and S. Lecoeuche, “A numerical solution to the minimum-time control problem for linear discrete-time systems,” *CoRR*, 2011

[3] J. V. Outrata, “On the minimum time problem in linear discrete systems with the discrete set of admissible controls,” *Kybernetika*, vol. 11, no. 5, pp. 368–374, 1975.

Georgia Tech
CREATING THE NEXT

Thank you for this opportunity!

Backup Slide: Model Details, Energy Removal Perspective