Dual-Domain Image Denoising

ECE 6500: *Fourier Techniques and Signal Analysis*

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Presentation Outline

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   - Types of Noise
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Motivating The Denoising Problem

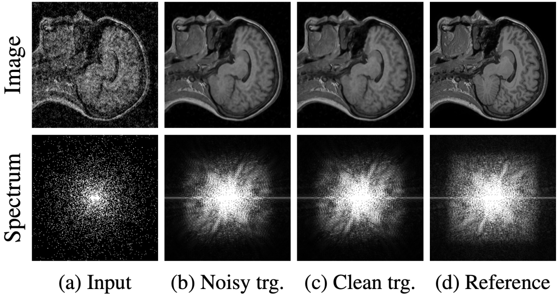

Figure: Recovering critical details from corrupted MRI images [2]

- Image de-noising algorithms attempt to recover the original image in the presence of noise.
- Widely applicable: Surveillance, Medicine, Biology, Film

Various Types of Noise

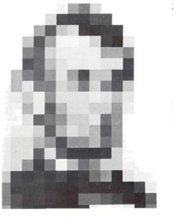

Figure: Quantization Noise [1]

Figure: Salt & Pepper Noise [2]

Figure: Periodic Noise [2]

- **Quantization Noise**, **Anisotropic Noise**, **Periodic Noise**, **Salt and Pepper Noise** - Straightforward solutions for these in isolation
- **Gaussian Noise** - Most commonly treated in the literature. Popular because there are generally multiple sources of random noise (CLT)
- We consider additive, random noise: $y = x + \eta$
  - Model η as stationary with known $\text{Var}[\eta] = \sigma$
  - y: Observed Image, x: True Image

[1] **Image Source:** `http://dada.compart-bremen.de/item/artwork/508`

[2] **Image Source:** `https://www.quora.com/How-can-we-estimate-noise-from-an-image`

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Why Dual-Domain?

A Fundamental Tradeoff

It is desirable to preserve edges and low contrast textures in images, while removing noise.

	Spatial Domain Techniques	Transform Domain Techniques
Preserve Sharp Edges?	✓	✗
Preserve Textures?	✗	✓

Want To Preserve Edges

- Filter in the *spatial domain*
- *Bilateral Filtering* is a well known technique!

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Spatial Domain: The Bilateral Filter

Warm Up: The Gaussian Blur

Let $\mathcal{N}_{\mathbf{p}}$ be a square patch of an image, $\mathcal{I}$, centered about pixel $\mathbf{p}$.

$$GB_{\mathbf{p}} = \sum_{\mathbf{q}\in\mathcal{N}_{\mathbf{p}}} G_{\sigma_s}(||\mathbf{p}-\mathbf{q}||)\,\mathcal{I}_{\mathbf{q}}, \text{ where } G_{\sigma_s}(x) = \frac{1}{2\pi\sigma_s^2}\exp\left(\frac{x^2}{2\sigma_s^2}\right)$$

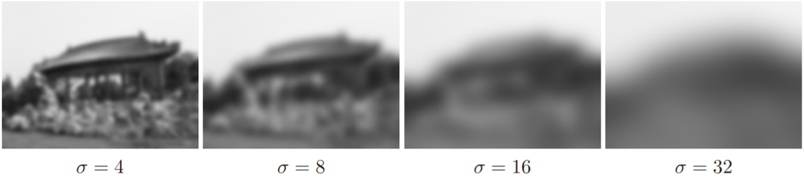

Figure: Gaussian blur with varying values of σ [3]

Spatial Domain: The Bilateral Filter

The Bilateral Filter

The Bilateral Filter introduces a second Gaussian kernel, which penalizes large differences in intensity:

$$BF_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} G_{\sigma_s}(||\mathbf{p} - \mathbf{q}||) G_{\sigma_r}(\mathcal{I}_{\mathbf{p}} - \mathcal{I}_{\mathbf{q}})\, \mathcal{I}_{\mathbf{q}}$$

Figure: Output and residual of bilateral filter [3]

Extension: The Joint/Guided Bilateral Filter

The Joint/Guided Bilateral Filter

For improved performance on low contrast images, the Bilateral Filter can be *guided* by a low-noise *guide image*, $\mathcal{G}$ by modifying the argument of the range kernel.

$$JBF_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} G_{\sigma_s}(||\mathbf{p} - \mathbf{q}||) G_{\sigma_r}(\underbrace{\mathcal{G}_{\mathbf{p}} - \mathcal{G}_{\mathbf{q}}}_{Formerly\ \mathcal{I}_{\mathbf{p}} - \mathcal{I}_{\mathbf{q}}}) \mathcal{I}_{\mathbf{q}}$$

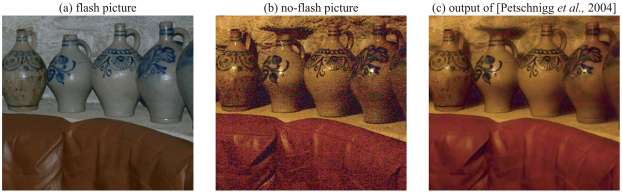

Figure: A scenario in which guided filtering is useful [4]

Domain Transform: STFT / Gabor Transform

The Big Picture

- The Short-Time (Discrete) Fourier Transform (STFT) is a Discrete Fourier Transform (DFT) performed on a *windowed* signal.
- If the windowing function is chosen as a Gaussian, then the overall transformation is known as the *Gabor Transform*.
- Sharp edges (high contrast spatial features) give rise to undesirable artifacts in the transform domain.

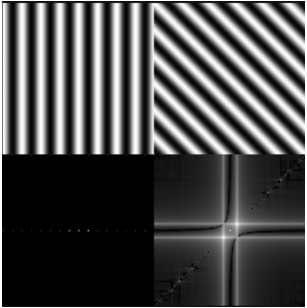

Figure: Illustration of frequency in images [3].

[3] **Image Source:** `https://www.cs.unm.edu/ brayer/vision/basis.gif`

Frequency Domain: Wavelet Shrinkage

Key Observation

- *The Fourier Coefficients of a noisy image will also contain noise.*

Effect of Noise on Fourier Coefficients

- The variance of the Fourier Coefficients in terms of the *known* variance of the noise $\mathrm{Var}[\eta] = \sigma$ (Slide 4) is given by [1]:

$$\sigma^2_{\mathbf{p},f} = \sigma^2 \sum_{\mathbf{q} \in \mathcal{N}_\mathbf{p}} \underbrace{k^2_{\mathbf{p},\mathbf{q}}}_{>0} > \sigma^2$$

- $k_{\mathbf{p},\mathbf{q}}$ is the product of two Gaussians: a *spatial* and a *range kernel*.

Frequency Domain: Wavelet Shrinkage

Key Observation

- *The Fourier Coefficients of a noisy image will also contain noise.*
  - *Shrink them!*

How Much To Shrink?

- Consider shrinkage factors $K_{\mathbf{p},f}$ as in [1]:

$$K_{\mathbf{p},f} = \exp\left(-\frac{\gamma_f \sigma^2_{\mathbf{p},f}}{|G_{\mathbf{p},f}|^2}\right) \quad \text{Note:} \lim_{|G_{\mathbf{p},f}|\to\infty} (K_{\mathbf{p},f}) = 1$$

where $G_{\mathbf{p},f}$ is the Gabor coefficient for frequency f in a patch, $\mathcal{F}_{\mathbf{p}}$, about pixel $\mathbf{p}$ in guide image $\mathcal{G}$.

Frequency Domain: Wavelet Shrinkage

Inverse DFT At The Origin, p

$$\tilde{\mathcal{I}}_{\mathbf{p}} = \frac{1}{|\mathcal{F}_{\mathbf{p}}|} \sum_{f \in \mathcal{F}_{\mathbf{p}}} K_{\mathbf{p},f} I_{\mathbf{p},f}$$

where $I_{\mathbf{p},f}$ is the Gabor coefficient for frequency f in a patch, $\mathcal{F}_{\mathbf{p}}$, about pixel **p** in original image $\mathcal{I}$.

How Much To Shrink?

- Recall:

$$K_{\mathbf{p},f} = \exp\left(-\frac{\gamma_f \sigma^2_{\mathbf{p},f}}{|G_{\mathbf{p},f}|^2}\right) \quad \text{Note:} \lim_{|G_{\mathbf{p},f}| \to \infty} (K_{\mathbf{p},f}) = 1$$

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Process Overview

DDID Process

The noisy image, $\mathcal{I}$, is first split into patches about each pixel $\mathbf{p}$. Each patch, $\mathcal{N}_\mathbf{p}$, is processed to yield a single, *denoised* pixel, $\tilde{\mathcal{I}}_\mathbf{p}$.

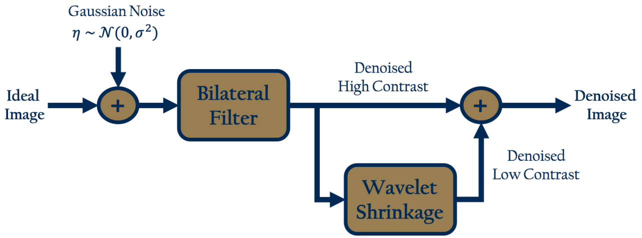

Figure: Block diagram of the Dual-Domain Image Denoising Process in [1]

Results and Performance

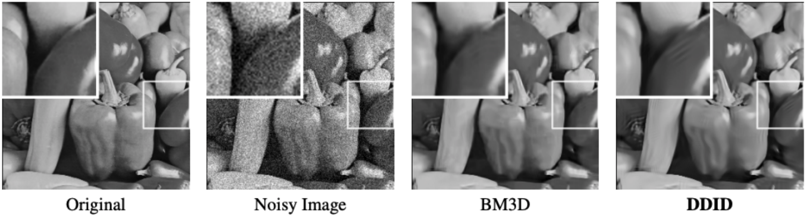

Figure: Comparison of DDID with other popular denoising techniques [1]

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